Mathematical problem writing

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Alex Neill (2008)

Understanding can be assessed by asking students to write their own mathematical problems in response to scenarios or prompts from their teachers.

When to use

Problem writing can be used at any time during a unit of work. It can cover ideas ranging from surface level knowledge through to deeper understandings. It can be useful for:

- assessing knowledge
- assessing understanding
- accessing existing ideas at the start of a unit of work
- uncovering common misconceptions
- stimulating discussion when used as a group task
- checking learning and deciding on next steps during a unit of work
- reviewing learning at the end of a unit of work
- peer assessment, either as students evaluate if the problem from another student fits the criteria laid down for it, or if another student can solve the problem

The theory

Constructivist theories of learning consider that students' existing understandings should be considered when developing teaching and learning programmes.

How the strategy works

When a student writes a problem of their own it helps uncover what they know, understand, and value in the particular mathematical topic to which their problem relates. This topic could be addition, subtraction, multiplication, division, estimation, or one of many other maths concepts.

What to do

1. Introduce the students to the specific area in which they will be asked to write a problem.
2. Specify the precise kind of problem that the student should write.
   Examples

   - "Write a word problem for the equation $4 + \square = 13$"
   - "Write a word problem for the equation $18 \times \frac{1}{2}$"
   - "Write a word problem that uses the concept of what happens to a number when it is multiplied by 10."
   - "Write a problem that uses the averaging method of estimation."
3. Give the students time to write their problems. This may also require time to conference students or to get them to edit their work.
4. Get the students to discuss and solve each other's problems. They should solve the problem or answer the question posed and state whether the problem or question is appropriate, and why or why not.
5. Give feedback to students or encourage them to give feedback to each other. Utilise what is uncovered in further teaching and learning.

A complete cycle of the teacher posing an original problem, students discussing it,

through to them writing and solving each other's problems is discussed in Estimation exposed (Neill, 2005).

Assessing students problems

When interpreting the problems students have written, look for:

- The **appropriateness** of how the question meets the criteria. Inappropriate questions may well indicate a lack of understanding.
  Example:
  If the question asks for a word problem that represents 18 ÷ ½, then
      "What is half of 18 apples?" (which is 18 × ½, and illustrates a confusion between division and multiplication).
- The level of **sophistication** of the question. Is it a simple knowledge question, or does it display deeper understanding? If students write relatively low level problems, ask them to write more challenging problems.
  Examples: (In ascending order of sophistication).

  1. Write a problem involving multiplying a number by 10.
         "What is 10 × 3?" (a simple knowledge-based response).
         "10 people each have 3 sweets. How many sweets are there altogether?"
         "What happens to any number in the forties when it is multiplied by 10?
         "What happens to decimal numbers multiplied by 10?"
  2. Write a problem about what happens when you add 0 on to numbers.
         "What is 57 + 0?"
         "What is 38 - 38 + 57?"|
         "What happens to any number when you add 0 to it?"
- Has the writer given **all necessary information** that the person needs to solve their problem? For example, if the task is to write a problem about calculating the circumference of a circle, is the diameter of the circle given (or deducible)?
- Can the student supply a **correct model answer**? The resource **Seed and cards (NM1224)** indicates that more able students tend to do this.
- Is the scenario the student creates posed as a **question** rather than as a statement?

Limitations

- A student's ability with written language may interfere with the mathematics being assessed. This difficulty with language is also seen when students need to write extended answers to mathematics problems. Potential language issues within mathematics are indicated in Neill (2000). Allow students to give their problems orally. Another approach is to incorporate the writing into a language session where students are reviewing, refining, conferencing, and editing by themselves, with peers or with the teacher.
- Some areas are more suitable than others for student problem writing.
  An example of this is **Estimating in Sport (NM1206)** a resource specifically about the averaging method of estimation. This would be suitable to assess student understanding about the averaging method because the numbers they select for their problem indicate their level of understanding of the technique. A resource about front-end estimation (e.g., **Estimating sums of money, NM1202**) would be less suitable as any numbers chosen by the student could use front-end method.

Adapting the strategy

- Get students to ask specific knowledge questions. This may lead on to developing higher level questions or posing more sophisticated problems.
- Using **halving and doubling (NM1246)**: students could be asked to write a problem

where doubling and halving will make the problem easier to solve.
- This strategy can be used in many other subject areas, including Science and English. For example, students could design questions about comprehension or inference (rather than about retrieving information) based on a text they have read. In Science, students could pose a set of questions that need to be answered after carrying out an experiment or practical task.

Further reading

- Barlow, A. T., & Drake, J. M. (2008) Division by a fraction: Assessing understanding through problem writing. Mathematics teaching in the middle schools, 13 (6), 326-332.
- Harrell, C. P., (2003) Writing in mathematics: A powerful tool to support math learning. Math counts: Issues that matter. Macmillan McGraw-Hill. Retrieved 17 March 2008, from www.mhschool.com/math/2003/teacher/teachres/mathissues/pdfs/math_writing....
- Neill, A. (2005) Estimation exposed. set: Research information for teachers, 1, 48-53.
- Neill, A. (2000) Diagnosing misconceptions in mathematics: Using the Assessment Resource Banks to remedy student errors. set: Research information for teachers, 1, 40-45.

Examples of ARB resources that use problem writing

Resource List

- Vehicle weights
- Friday Algebra
- Age and time problems
- Adding and subtracting
- What's my number?
- Writing word problems
- Selling cars
- Making number sentences
- Seeds and cards
- Making number sentences II
- Number sentences and story problems