Some maths problems – Analysis of student responses

Link to the assessment resource, *Some maths problems* (NM1333)

Table 1: Frequency of use and success rates of different strategies

Strategy	a) 154 + 38	b) 357 + 162	c) 326 + 279	d) 736 + 589	Total
Partitions to jump through tidy numbers	17 (13) 76%	26 (15) 58%	25 (15) 60%	29 (14) 48%	97 (57) 59%
Partitions with tidy numbers	14 (12) 86%	7 (4) 57%	13 (10) 77%	9 (6) 67%	43 (32) 74%
PV partitions just one number	30 (24) 83%	16 (11) 69%	12 (9) 75%	11 (6) 55%	69 (50) 72%
PV partitions both numbers	76 (67) 88%	87 (59) 68%	78 (61) 78%	69 (44) 64%	310 (231) 75%
Vertical algorithm or equivalent	43 (34) 79%	41 (27) 66%	45 (27) 60%	47 (32) 68%	176 (120) 68%
ALL STRATEGIES	**180 (150) 83%**	**177 (116) 66%**	**173 (122) 71%**	**165 (102) 62%**	**695 (490) 71%**
States answer	3 (2)	4 (2)	5 (3)	5 (2)	17 (9) 53%
Subtract rather than add	4 (0)	4 (0)	4 (0)	3 (0)	15 (0) 0%
Other statements	7 (1)	6 (1)	4 (0)	8 (0)	25 (2) 8%
Missing	7 (4)	10 (4)	15 (1)	20 (1)	52 (10) 19%
TOTAL	**201 (157) 78%**	**201 (123) 61%**	**201 (126) 63%**	**201 (105) 52%**	**804 (511) 64%**

Based on a representative sample of *201* students

Table 2: Mean abilities of students using different strategies

Strategy	a) 154 + 38	b) 357 + 162	c) 326 + 279	d) 736 + 589	Total
Partitions to jump through tidy numbers	23.9	24.0	22.8	24.7	24.4
Partitions with tidy numbers	22.0	25.9	26.8	24.8	24.7
PV partitions just one number	25.8	22.7	21.8	22.3	23.8
PV partitions both numbers	23.5	24.3	25.1	25.2	24.5
Vertical algorithm or equivalent	19.4	19.2	18.6	19.9	19.3
States answer	15.3	16.5	18.4	17.4	17.1
Subtract rather than add	8.8	8.8	14.0	9.3	10.3
Other statements	15.1	14.5	17.0	17.9	16.2
Missing	11.0	10.2	12.0	12.8	11.6

Based on a representative sample of *201* students

* Mean ability – average score out of 43 of all students using this strategy on a test set of seven questions.

** [Σ (mean ability × number using strategy in each part of the question)] / total using the strategy in any part of the question

e.g., for Partitions to jump through tidy numbers =

$$[(23.9 \times 17) + (14.0 \times 26) + (22.8 \times 25) + (24.7 \times 29)] \div 97 = 24.4$$

Patterns within the strategies used

Students who used any of the four partitioning strategies mentioned below had approximately equal mean abilities when averaged over the four parts of the question. Students who used the vertical algorithm met with roughly equal success rates, but were of a markedly lower mean ability.

1. *Partitioning using rounding and compensation to jump through tidy numbers* was notably less successful than other strategies (including the vertical algorithm), with only 59% of students arriving at a correct answer, compared with 72 - 75% success rates using the other three strategies. This may be due to the extra complexity of keeping track of the various compensations.
2. *Partitioning by rounding one number to a tidy number then compensation* was relatively more successful in parts a) and c), where just the tens boundary was crossed.
3. *Place value partitioning one number into hundreds, tens and ones and adding it on to the other in parts* was used far more often in part a) where the second number just needed to be partitioned into *tens* and *ones*. Students who used it for parts b) – d) had somewhat lower mean abilities than those using it in a). Students almost exclusively partitioned the second number into *hundreds*, *tens* and *ones* and added these on to the first number.
4. *Place value partitioning both numbers in hundreds, tens, and ones* was by far the most common strategy in all parts and was consistently successful. It may be that for adding larger numbers, it is easier to keep track of the number of *tens*, *hundreds* and *thousands*.
5. *Place value partitioning expressing tens or hundreds* **as** *ones* [e.g., 357 + 162 = (3 + 1) *lots of 100* plus (5 + 6) *lots of 10* plus (7 + 2)] was used by students with lower mean ability than those who expressed number as *tens* or *hundreds*.

Mental strategies:

Many students who used the above strategies, showed only part of their working, particularly in parts b) – d). They were clearly doing some two-digit calculations mentally. For example:

(b) 357 + 162 = (350 + 160) + (7 + 2) **or** (300 +100) + (57 + 62)
(c) 326 + 270 = 596; 596 + 9 = 605
(d) 740 + 590 = 1330; 1330 – 4 – 1 = 1325

These students had roughly equal mean abilities as those who did not use two-digit mental arithmetic, but had somewhat lower success rates due to errors in their mental arithmetic.